Mark Scheme 4725 January 2007

\begin{tabular}{|c|c|c|c|c|}
\hline 1. \& \begin{tabular}{l}
(i) \(a=-3\) \\
(ii)
\[
\begin{aligned}
\& 2 a-3=7 \text { or } 3 a-6=9 \\
\& a=5
\end{aligned}
\]
\end{tabular} \& \[
\begin{array}{|l}
\hline \text { B1 } \\
\text { M1 } \\
\text { A1 }
\end{array}
\] \& 1

2
3 \& State correct value Sensible attempt at multiplication Obtain correct answer \\

\hline 2. \& $$
x^{2}-y^{2}=15 \text { and } x y=4
$$

$$
\pm(4+i)
$$ \& M1

A1 A1
M1
DM1
A1 \& 6

6 \& | Attempt to equate real and imaginary parts of $(x+\mathrm{i} y)^{2}$ and 15 $+8 \mathrm{i}$ |
| :--- |
| Obtain each result |
| Eliminate to obtain a quadratic in $x^{2}$ or $y^{2}$ |
| Solve to obtain $x=( \pm) 4$, or $y=$ $( \pm) 1$ |
| Obtain only correct two answers as complex numbers | \\

\hline 3. \& $$
\frac{1}{4} n^{2}(n+1)^{2}-\frac{1}{2} n(n+1)
$$

$$
\frac{1}{4} n(n-1)(n+1)(n+2)
$$ \& \[

$$
\begin{aligned}
& \text { M1 } \\
& \text { M1 } \\
& \text { A1 } \\
& \text { M1 } \\
& \text { A1 } \\
& \text { A1 }
\end{aligned}
$$
\] \& 6

6 \& | Expand to obtain $r^{3}-r$ |
| :--- |
| Consider difference of two standard results |
| Obtain correct unfactorised answer |
| Attempt to factorise |
| Obtain factor of $\frac{1}{4} n(n+1)$ |
| Obtain correct answer | \\

\hline 4. \& | (i) |
| :--- |
| (ii) | \& \[

$$
\begin{aligned}
& \text { B1 } \\
& \text { B1 } \\
& \text { B1 } \\
& \text { B1 } \\
& \text { B1 } \\
& \text { B1 }
\end{aligned}
$$

\] \& 3 \& | Circle |
| :--- |
| Centre ( $1,-1$ ) |
| Passing through $(0,0)$ |
| Sketch a concentric circle |
| Inside (i) and touching axes |
| Shade between the circles | \\

\hline 5. \& (i) \& B1 \& 1 \& Show given answer correctly \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|}
\hline \& \begin{tabular}{l}
(ii)
\[
-1 \pm \mathrm{i} \sqrt{3}
\] \\
(iii)
\end{tabular} \& \[
\begin{array}{|l}
\hline \text { M1 } \\
\text { A1 } \\
\text { A1 } \\
\text { B1 } \\
\text { B1 } \\
\text { B1 }
\end{array}
\] \& 3

3

7 \& | Attempt to solve quadratic equation or substitute $x+\mathrm{i} y$ and equate real and imaginary parts |
| :--- |
| Obtain answers as complex numbers Obtain correct answers, simplified Correct root on $x$ axis, co-ords. shown |
| Other roots in $2^{\text {nd }}$ and $3^{\text {rd }}$ quadrants |
| Correct lengths and angles or coordinates or complex numbers shown | \\

\hline 6. \& | (i) $u_{n+1}-u_{n}=2 n+4$ |
| :--- |
| (ii) | \& \[

$$
\begin{array}{|l}
\hline \text { B1 } \\
\text { M1 } \\
\text { A1 } \\
\text { B1 } \\
\text { M1 } \\
\text { M1 } \\
\text { A1 } \\
\text { A1 }
\end{array}
$$

\] \& 5 \& | Correct expression for $u_{n+1}$ |
| :--- |
| Attempt to expand and simplify |
| Obtain given answer correctly |
| State $u_{1}=4$ ( or $u_{2}=10$ ) and is divisible by 2 |
| State induction hypothesis true for |
| $u_{n}$ |
| Attempt to use result in (ii) |
| Correct conclusion reached for $u_{n+1}$ |
| Clear,explicit statement of induction conclusion | \\


\hline 7. \& | (i) $\alpha+\beta=-5 \quad \alpha \beta=10$ |
| :--- |
| (ii) $\alpha^{2}+\beta^{2}=5$ |
| (iii) $x^{2}-\frac{1}{2} x+1=0$ | \& B1 B1

M1
A1
B1
M1
A1
B1ft \& 2
2

4

8 \& | State correct values |
| :--- |
| Use $(\alpha+\beta)^{2}-2 \alpha \beta$ |
| Obtain given answer correctly, using value of -5 |
| Product of roots $=1$ |
| Attempt to find sum of roots |
| Obtain $\frac{5}{10}$ or equivalent |
| Write down required quadratic equation, or any multiple. | \\

\hline
\end{tabular}

| 8. | (i) $(r+1)^{2} r!$ <br> (ii) $(n+2)!-2!$ <br> (iii) | M1 <br> A1 <br> A1 <br> M1 <br> A1 <br> M1 <br> A1 <br> B1ft | 4 1 | Factor of $r$ ! or $(\mathrm{r}+1)$ ! seen <br> Factor of $(r+1)$ found <br> Obtain given answer correctly <br> Express terms as differences using <br> (i) <br> At least $1^{\text {st }}$ two and last term correct <br> Show that pairs of terms cancel <br> Obtain correct answer in any form <br> Convincing statement for nonconverging, ft their (ii) |
| :---: | :---: | :---: | :---: | :---: |
| 9. | (i) $\binom{0}{0}\binom{0}{-1}\binom{3}{0}\binom{3}{-1}$ |  | 2 | For at least two correct images <br> For correct diagram, co-ords.clearly written down |
|  | (ii) $90^{\circ}$ clockwise, centre origin $\left(\begin{array}{cc} 0 & 1 \\ -10 \end{array}\right)$ | B1 B1 <br> B1 | 3 | Or equivalent correct description <br> Correct matrix, not in trig form |
|  | (iii) Stretch parallel to $x$-axis, s.f. 3 $\left(\begin{array}{ll} 3 & 0 \\ 0 & 1 \end{array}\right)$ | B1 B1 <br> B1 B1 | 4 | Or equivalent correct description, but must be a stretch for $2^{\text {nd }} \mathrm{B} 1$ <br> Each correct column |


| 10. | (i) $\Delta=\operatorname{det} \mathbf{D}=3 a-6$ $\mathbf{D}^{-1}=\frac{1}{\Delta}\left(\begin{array}{rrr} 3 & -2 & 4 \\ -3 & a & -2 a \\ -3 & a & a-6 \end{array}\right)$ <br> (ii) $\frac{1}{\Delta}\left(\begin{array}{r}5 \\ 2 a-9 \\ 5 a-15\end{array}\right)$ | M1 M1 A1 M1 A1 B1 A1 M1 A1A1A1 ft all 3 | 7 4 4 11 | Show correct expansion process for $3 \times 3$ <br> Correct evaluation of any $2 \times 2$ det <br> Obtain correct answer <br> Show correct process for adjoint entries <br> Obtain at least 4 correct entries in adjoint <br> Divide by their determinant <br> Obtain completely correct answer <br> Attempt product of form $\mathbf{D}^{-1} \mathbf{C}$, or eliminate to get 2 equations and solve <br> Obtain correct answers, ft their inverse |
| :---: | :---: | :---: | :---: | :---: |

